Ideal pseudointersection numbers

Jaroslav Šupina

Institute of Mathematics
Faculty of Science
P.J. Šafárik University in Košice

26^{th} of January 2020

Pseudointersection numbers

- $\mathfrak{p}$ (F. Rothberger 1948)

Pseudointersection numbers

- $\mathfrak{p}$ (F. Rothberger 1948)
- $\mathtt{cov}^*(\mathcal{I})$ (J. Brendle and S. Shelah 1999, $\pi\mathfrak{p}(\mathcal{U})$
  F. Hernández and M. Hrušák 2007)

Pseudointersection numbers

- $\mathfrak{p}$ (F. Rothberger 1948)
- $\mathtt{cov}^*(\mathcal{I})$ (J. Brendle and S. Shelah 1999, $\pi\mathfrak{p}(\mathcal{U})$
  F. Hernández and M. Hrušák 2007)
- $\mathfrak{p}_\square(\mathcal{J})$ (P. Borodulin–Nadzieja and B. Farkas 2012)

Pseudointersection numbers

- $\mathfrak{p}$ (F. Rothberger 1948)
- $\mathrm{cov}^*(\mathcal{I})$ (J. Brendle and S. Shelah 1999, $\pi\mathfrak{p}(\mathcal{U})$
  F. Hernández and M. Hrušák 2007)
- $\mathfrak{p}_{\square}(\mathcal{J})$ (P. Borodulin–Nadzieja and B. Farkas 2012)
- $\mathfrak{p}_{\square}(\mathcal{I},\mathcal{J})$ (M. Repický 2018)

Pseudointersection numbers

- $\mathfrak{p}$ (F. Rothberger 1948)
- $\mathtt{cov}^*(\mathcal{I})$ (J. Brendle and S. Shelah 1999, $\pi\mathfrak{p}(\mathcal{U})$
  F. Hernández and M. Hrušák 2007)
- $\mathfrak{p}_\square(\mathcal{J})$ (P. Borodulin–Nadzieja and B. Farkas 2012)
- $\mathfrak{p}_\square(\mathcal{I},\mathcal{J})$ (M. Repický 2018)
- $\lambda(\triangle,\nabla)$ (V. Šottová and J.Š.)

The role of $\mathtt{cov}^*(\mathcal{I})$

The role of $\mathtt{cov}^*(\mathcal{I})$

- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \leq \mathfrak{b}$ (P. Nyikos)

The role of $\mathtt{cov}^*(\mathcal{I})$

- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \leq \mathfrak{b}$ (P. Nyikos)
- $\min\{\mathtt{cov}^*(\mathcal{I}), \mathfrak{b}\} = \lambda(\mathcal{I}, \mathrm{Fin})$ (V. Šottová and J.Š. 2019)

The role of $\mathrm{cov}^*(\mathcal{I})$

- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \leq \mathfrak{b}$ (P. Nyikos)
- $\min\{\mathtt{cov}^*(\mathcal{I}), \mathfrak{b}\} = \lambda(\mathcal{I}, \mathrm{Fin})$ (V. Šottová and J.Š. 2019)
- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{p}_{\mathrm{K}}(\mathcal{J})\} = \mathfrak{p}$

The role of $\mathtt{cov}^*(\mathcal{I})$

- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \leq \mathfrak{b}$ (P. Nyikos)
- $\min\{\mathtt{cov}^*(\mathcal{I}), \mathfrak{b}\} = \lambda(\mathcal{I}, \mathrm{Fin})$ (V. Šottová and J.Š. 2019)
- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{p}_{\mathrm{K}}(\mathcal{J})\} = \mathfrak{p}$
- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{p}_{\mathrm{K}}(\mathcal{I}, \mathcal{J})\} = \mathtt{cov}^*(\mathcal{I})$

The role of $\mathtt{cov}^*(\mathcal{I})$

- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \leq \mathfrak{b}$ (P. Nyikos)
- $\min\{\mathtt{cov}^*(\mathcal{I}), \mathfrak{b}\} = \lambda(\mathcal{I}, \mathrm{Fin})$ (V. Šottová and J.Š. 2019)
- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{p}_{\mathrm{K}}(\mathcal{J})\} = \mathfrak{p}$
- $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{p}_{\mathrm{K}}(\mathcal{I}, \mathcal{J})\} = \mathtt{cov}^*(\mathcal{I})$
- $\min\{\mathtt{cov}^*(\mathcal{I}), \mathfrak{b}_{\mathcal{J}}\} \leq \lambda(\mathcal{I}, \mathcal{J}) \leq \mathfrak{b}_{\mathcal{J}}$

Problem 1: countable $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

$C_p(X)$ has countable Fréchet-Urysohn property if and only if $C_p(X)$ is an $\mathrm{S}_1(\Omega_{\mathbf{0}}^{\mathtt{ct}}, \Gamma_{\mathbf{0}})$-space.

Problem 1: countable $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

$C_p(X)$ has countable Fréchet-Urysohn property if and only if $C_p(X)$ is an $\mathrm{S}_1(\Omega_{\mathbf{0}}^{\mathtt{ct}}, \Gamma_{\mathbf{0}})$-space.

P. Borodulin-Nadzieja and B. Farkas 2012

$\mathcal{I}$-Fréchet-Urysohn property

Problem 1: countable $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

$C_p(X)$ has countable Fréchet-Urysohn property if and only if $C_p(X)$ is an $\mathrm{S}_1(\Omega_{\mathbf{0}}^{\mathtt{ct}}, \Gamma_{\mathbf{0}})$-space.

P. Borodulin-Nadzieja and B. Farkas 2012

$\mathcal{I}$-Fréchet-Urysohn property

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH**:

Problem 1: countable $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

$C_p(X)$ has countable Fréchet-Urysohn property if and only if $C_p(X)$ is an $\mathrm{S}_1(\Omega_{\mathbf{0}}^{\mathtt{ct}}, \Gamma_{\mathbf{0}})$-space.

P. Borodulin-Nadzieja and B. Farkas 2012

$\mathcal{I}$-Fréchet-Urysohn property

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH**:

- there is a meager ideal $\mathcal{I}$,
- there is a set of reals A of size ω_1,

Problem 1: countable $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

$C_p(X)$ has countable Fréchet-Urysohn property if and only if $C_p(X)$ is an $\mathrm{S}_1(\Omega_{\mathbf{0}}^{\mathtt{ct}}, \Gamma_{\mathbf{0}})$-space.

P. Borodulin-Nadzieja and B. Farkas 2012

$\mathcal{I}$-Fréchet-Urysohn property

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH**:

- there is a meager ideal $\mathcal{I}$,
- there is a set of reals A of size ω_1,
- $C_p(A)$ has countable $\mathcal{I}$-Fréchet-Urysohn property,
- $C_p(A)$ is not an $\mathrm{S}_1(\Omega_{\mathbf{0}}^{\mathtt{ct}}, \mathcal{I}\text{-}\Gamma_{\mathbf{0}})$-space.

Problem 2: covering counterpart of $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

X is an $\mathrm{S}_1(\Omega^{\mathtt{ct}}, \Gamma)$-space if and only if X has $\binom{\Omega^{\mathtt{ct}}}{\Gamma}$.

Problem 2: covering counterpart of $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

X is an $\mathrm{S}_1(\Omega^{\mathtt{ct}}, \Gamma)$-space if and only if X has $\binom{\Omega^{\mathtt{ct}}}{\Gamma}$.

Question (B. Tsaban ESTC 2019, Vienna)

Is it true that X is an $\mathrm{S}_1(\Omega^{\mathtt{ct}}, \mathcal{I}\text{-}\Gamma)$-space if and only if X has $\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{I}\text{-}\Gamma\end{smallmatrix}\right]$?

Problem 2: covering counterpart of $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

X is an $\mathrm{S}_1(\Omega^{\mathrm{ct}}, \Gamma)$-space if and only if X has $\binom{\Omega^{\mathrm{ct}}}{\Gamma}$.

Question (B. Tsaban ESTC 2019, Vienna)

Is it true that X is an $\mathrm{S}_1(\Omega^{\mathrm{ct}}, \mathcal{I}\text{-}\Gamma)$-space if and only if X has $\left[{\Omega^{\mathrm{ct}} \atop \mathcal{I}\text{-}\Gamma}\right]$?

P. Borodulin-Nadzieja and B. Farkas 2012

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH**:

- there is a meager ideal $\mathcal{I}$,
- there is a set of reals A of size ω_1,
- A has $\left[{\Omega^{\mathrm{ct}} \atop \mathcal{I}\text{-}\Gamma}\right]$,
- A is not an $\mathrm{S}_1(\Omega^{\mathrm{ct}}, \mathcal{I}\text{-}\Gamma)$-space.

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ *and* $\mathfrak{p}_{\text{1-1}}(\mathcal{J})$*?*

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ *and* $\mathfrak{p}_{\text{1-1}}(\mathcal{J})$*?*

$$\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\mathrm{KB}}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\text{1-1}}) = \mathfrak{p}_{\text{1-1}}(\mathcal{J})$$

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ *and* $\mathfrak{p}_{\text{1-1}}(\mathcal{J})$*?*

$$\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\mathrm{KB}}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\text{1-1}}) = \mathfrak{p}_{\text{1-1}}(\mathcal{J})$$

Different repetitions of elements (infinitely many, finitely many, none) in the enumeration of sequence.

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ *and* $\mathfrak{p}_{\text{1-1}}(\mathcal{J})$*?*

$$\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\mathrm{KB}}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\text{1-1}}) = \mathfrak{p}_{\text{1-1}}(\mathcal{J})$$

Different repetitions of elements (infinitely many, finitely many, none) in the enumeration of sequence.

Similarly for functional versions.

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ *and* $\mathfrak{p}_{1\text{-}1}(\mathcal{J})$*?*

$$\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\mathrm{KB}}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{1\text{-}1}) = \mathfrak{p}_{1\text{-}1}(\mathcal{J})$$

Different repetitions of elements (infinitely many, finitely many, none) in the enumeration of sequence.

Similarly for functional versions.

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Is $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$ *for each analytic (*P*-)ideal* $\mathcal{J}$*?*

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ *and* $\mathfrak{p}_{\text{1-1}}(\mathcal{J})$*?*

$$\mathtt{non}\left(\begin{bmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{bmatrix}\right) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \qquad \mathtt{non}\left(\begin{bmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{bmatrix}_{\mathrm{KB}}\right) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \qquad \mathtt{non}\left(\begin{bmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{bmatrix}_{\text{1-1}}\right) = \mathfrak{p}_{\text{1-1}}(\mathcal{J})$$

Different repetitions of elements (infinitely many, finitely many, none) in the enumeration of sequence.

Similarly for functional versions.

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Is $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$ *for each analytic (*P*-)ideal* $\mathcal{J}$*?*

Proposition

If $\mathcal{J}$ *is a meager* P*-ideal then* $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$*.*

Pseudointersection numbers $\mathfrak{p}$ and $\mathtt{cov}^*(\mathcal{I})$

$$\mathfrak{p} \quad = \quad \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \text{ does not have a pseudounion}\}$$

Pseudointersection numbers $\mathfrak{p}$ and $\mathtt{cov}^*(\mathcal{I})$

$$\mathfrak{p} = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \text{ does not have a pseudounion}\}$$

$$\mathtt{cov}^*(\mathcal{I}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{I} \wedge \mathcal{A} \text{ does not have a pseudounion}\}$$

Pseudointersection numbers $\mathfrak{p}$ and $\mathtt{cov}^*(\mathcal{I})$

$$\mathfrak{p} = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \text{ does not have a pseudounion}\}$$

$$\mathtt{cov}^*(\mathcal{I}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{I} \wedge \mathcal{A} \text{ does not have a pseudounion}\}$$

Convention: $\min \emptyset = +\infty$

Pseudointersection numbers $\mathfrak{p}$ and $\mathtt{cov}^*(\mathcal{I})$

$$\mathfrak{p} = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \text{ does not have a pseudounion}\}$$

$$\mathtt{cov}^*(\mathcal{I}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{I} \wedge \mathcal{A} \text{ does not have a pseudounion}\}$$

Convention: $\min \emptyset = +\infty$

Fin	Fin $\times$ Fin	$\mathtt{S}$	$\mathcal{ED}$	$\mathcal{R}$	$\mathtt{conv}$	$\mathtt{nwd}$
$+\infty$	$\mathfrak{b}$	$\mathtt{non}(\mathcal{N})$	$\mathtt{non}(\mathcal{M})$	$\mathfrak{c}$	$\mathfrak{c}$	$\mathtt{cov}(\mathcal{M})$

The inequality $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \leq \mathfrak{b}$

The inequality $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \leq \mathfrak{b}$

Observation

If X is an $[\mathcal{J}\text{-}\Gamma, \Gamma]$-space and an $\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma)$-space then X is an $\mathrm{S}_1(\Gamma, \Gamma)$-space.

The inequality $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \leq \mathfrak{b}$

Observation

If X is an $[\mathcal{J}\text{-}\Gamma, \Gamma]$-space and an $\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma)$-space then X is an $\mathrm{S}_1(\Gamma, \Gamma)$-space.

Corollary

$$\min\{\mathtt{non}([\mathcal{J}\text{-}\Gamma, \Gamma]), \mathtt{non}(\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma))\} \leq \mathtt{non}(\mathrm{S}_1(\Gamma, \Gamma))$$

The inequality $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \leq \mathfrak{b}$

Observation

If X is an $[\mathcal{J}\text{-}\Gamma, \Gamma]$-space and an $\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma)$-space then X is an $\mathrm{S}_1(\Gamma, \Gamma)$-space.

Corollary

$$\min\{\mathtt{non}([\mathcal{J}\text{-}\Gamma, \Gamma]), \mathtt{non}(\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma))\} \leq \mathtt{non}(\mathrm{S}_1(\Gamma, \Gamma))$$

- X is an $[\mathcal{J}\text{-}\Gamma, \Gamma]$-space if for every $\langle V_n : n \in \omega\rangle$ of $\mathcal{J}$-γ-covers there is $\varphi \in {}^{\omega}\omega$ such that $\langle V_{\varphi(m)} : m \in \omega\rangle$ is a γ-cover.

The inequality $\min\{\mathtt{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \le \mathfrak{b}$

Observation

If X is an $[\mathcal{J}\text{-}\Gamma, \Gamma]$-space and an $\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma)$-space then X is an $\mathrm{S}_1(\Gamma, \Gamma)$-space.

Corollary

$$\min\{\mathbf{non}([\mathcal{J}\text{-}\Gamma, \Gamma]), \mathbf{non}(\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma))\} \le \mathbf{non}(\mathrm{S}_1(\Gamma, \Gamma))$$

- X is an $[\mathcal{J}\text{-}\Gamma, \Gamma]$-space if for every $\langle V_n : n \in \omega\rangle$ of $\mathcal{J}$-γ-covers there is $\varphi \in {}^{\omega}\omega$ such that $\langle V_{\varphi(m)} : m \in \omega\rangle$ is a γ-cover.
- A sequence $\langle V_n : n \in \omega\rangle$ of open subsets of X such that $V_n \neq X$ is $\mathcal{J}$-γ-cover if $\{n : x \notin V_n\} \in \mathcal{J}$ for every $x \in V_n$.

The inequality $\min\{\mathrm{cov}^*(\mathcal{J}), \mathfrak{b}_{\mathcal{J}}\} \le \mathfrak{b}$

Observation

If X is an $[\mathcal{J}\text{-}\Gamma, \Gamma]$-space and an $\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma)$-space then X is an $\mathrm{S}_1(\Gamma, \Gamma)$-space.

Corollary

$$\min\{\mathbf{non}([\mathcal{J}\text{-}\Gamma, \Gamma]), \mathbf{non}(\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma))\} \le \mathbf{non}(\mathrm{S}_1(\Gamma, \Gamma))$$

- X is an $[\mathcal{J}\text{-}\Gamma, \Gamma]$-space if for every $\langle V_n : n \in \omega\rangle$ of $\mathcal{J}$-γ-covers there is $\varphi \in {}^\omega\omega$ such that $\langle V_{\varphi(m)} : m \in \omega\rangle$ is a γ-cover.
- A sequence $\langle V_n : n \in \omega\rangle$ of open subsets of X such that $V_n \neq X$ is $\mathcal{J}$-γ-cover if $\{n : x \notin V_n\} \in \mathcal{J}$ for every $x \in V_n$.
- X is an $\mathrm{S}_1(\Gamma, \mathcal{J}\text{-}\Gamma)$-space if for every $\langle\langle V_{n,m} : m \in \omega\rangle : n \in \omega\rangle$ of γ-covers there is $\varphi \in {}^\omega\omega$ such that $\langle V_{n,\varphi(n)} : n \in \omega\rangle$ is a $\mathcal{J}$-γ-cover.

Pseudointersection numbers $\mathfrak{p}_\square(\mathcal{J})$ and $\mathfrak{p}_\square(\mathcal{I}, \mathcal{J})$

$\square \in \{1\text{-}1, \mathrm{KB}, \mathrm{K}\}$

Pseudointersection numbers $\mathfrak{p}_\Box(\mathcal{J})$ and $\mathfrak{p}_\Box(\mathcal{I},\mathcal{J})$

$\Box \in \{\text{1-1}, \text{KB}, \text{K}\}$

$$\mathfrak{p} \quad = \quad \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \not\leq_\Box \text{Fin}\}$$

Pseudointersection numbers $\mathfrak{p}_\Box(\mathcal{J})$ and $\mathfrak{p}_\Box(\mathcal{I},\mathcal{J})$

$\Box \in \{1\text{-}1, \mathrm{KB}, \mathrm{K}\}$

$$\mathfrak{p} = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \not\leq_\Box \mathrm{Fin}\}$$

$$\mathfrak{p}_\Box(\mathcal{J}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \not\leq_\Box \mathcal{J}\}$$

Pseudointersection numbers $\mathfrak{p}_\Box(\mathcal{J})$ and $\mathfrak{p}_\Box(\mathcal{I},\mathcal{J})$

$\Box \in \{1\text{-}1, \mathrm{KB}, \mathrm{K}\}$

$$\mathfrak{p} = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \not\leq_\Box \mathrm{Fin}\}$$

$$\mathfrak{p}_\Box(\mathcal{J}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \not\leq_\Box \mathcal{J}\}$$

$$\mathfrak{p}_\Box(\mathcal{I},\mathcal{J}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{I} \wedge \mathcal{A} \not\leq_\Box \mathcal{J}\}$$

Pseudointersection numbers $\mathfrak{p}_\Box(\mathcal{J})$ and $\mathfrak{p}_\Box(\mathcal{I},\mathcal{J})$

$\Box \in \{\text{1-1}, \mathrm{KB}, \mathrm{K}\}$

$$\mathfrak{p} = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \not\leq_\Box \mathrm{Fin}\}$$

$$\mathfrak{p}_\Box(\mathcal{J}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \not\leq_\Box \mathcal{J}\}$$

$$\mathfrak{p}_\Box(\mathcal{I},\mathcal{J}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{I} \wedge \mathcal{A} \not\leq_\Box \mathcal{J}\}$$

$$\mathfrak{p}_\Box(\mathcal{J}) = \min\{\mathfrak{p}_\Box(\mathcal{I},\mathcal{J}) : \mathcal{I} \text{ is an ideal}\}$$

Pseudointersection numbers $\mathfrak{p}_\square(\mathcal{J})$ and $\mathfrak{p}_\square(\mathcal{I},\mathcal{J})$

$\square \in \{1\text{-}1, \mathrm{KB}, \mathrm{K}\}$

$$\mathfrak{p} = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \not\leq_\square \mathrm{Fin}\}$$

$$\mathfrak{p}_\square(\mathcal{J}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{P}(\omega) \text{ has fup } \wedge \mathcal{A} \not\leq_\square \mathcal{J}\}$$

$$\mathfrak{p}_\square(\mathcal{I},\mathcal{J}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{I} \wedge \mathcal{A} \not\leq_\square \mathcal{J}\}$$

$$\mathfrak{p}_\square(\mathcal{J}) = \min\{\mathfrak{p}_\square(\mathcal{I},\mathcal{J}) : \mathcal{I} \text{ is an ideal}\}$$

$$\begin{array}{ccccccc}
\mathtt{cov}^*(\mathcal{I}) & \longrightarrow & \mathfrak{p}_{1\text{-}1}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{I},\mathcal{J}) \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\mathfrak{p} & \longrightarrow & \mathfrak{p}_{1\text{-}1}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{J})
\end{array}$$

Pseudointersection numbers $\mathfrak{p}_{\square}(\mathcal{J})$ and $\mathfrak{p}_{\square}(\mathcal{I},\mathcal{J})$

$$
\begin{array}{ccccccc}
\mathtt{cov}^*(\mathcal{I}) & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{I},\mathcal{J}) \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\mathfrak{p} & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{J})
\end{array}
$$

Pseudointersection numbers $\mathfrak{p}_\square(\mathcal{J})$ and $\mathfrak{p}_\square(\mathcal{I},\mathcal{J})$

$$\begin{array}{ccccccc}
\mathtt{cov}^*(\mathcal{I}) & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{I},\mathcal{J}) \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\mathfrak{p} & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{J})
\end{array}$$

Theorem (P. Borodulin-Nadzieja and B. Farkas 2012)

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH** *the following hold.*

Pseudointersection numbers $\mathfrak{p}_\square(\mathcal{J})$ and $\mathfrak{p}_\square(\mathcal{I},\mathcal{J})$

$$\begin{array}{ccccccc}
\mathtt{cov}^*(\mathcal{I}) & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{I},\mathcal{J}) \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\mathfrak{p} & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{J})
\end{array}$$

Theorem (P. Borodulin-Nadzieja and B. Farkas 2012)

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH** *the following hold.*

(1) *There is a filter $\mathcal{F}$ with $\mathfrak{p}_{\text{1-1}}(\mathcal{F}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{F}) = \mathfrak{p}_{\mathrm{K}}(\mathcal{F}) = \omega_2$.*

Pseudointersection numbers $\mathfrak{p}_\square(\mathcal{J})$ and $\mathfrak{p}_\square(\mathcal{I},\mathcal{J})$

$$
\begin{array}{ccccccc}
\mathtt{cov}^*(\mathcal{I}) & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{I},\mathcal{J}) \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\mathfrak{p} & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{J})
\end{array}
$$

Theorem (P. Borodulin-Nadzieja and B. Farkas 2012)

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC+GCH** *the following hold.*

(1) *There is a filter $\mathcal{F}$ with $\mathfrak{p}_{\text{1-1}}(\mathcal{F}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{F}) = \mathfrak{p}_{\mathrm{K}}(\mathcal{F}) = \omega_2$.*

(2) *There is a meager filter $\mathcal{G}$ with $\mathfrak{p}_{\text{1-1}}(\mathcal{G}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{G}) = \omega_1$ and $\mathfrak{p}_{\mathrm{K}}(\mathcal{G}) = \omega_2$.*

Pseudointersection numbers $\mathfrak{p}_\square(\mathcal{J})$ and $\mathfrak{p}_\square(\mathcal{I},\mathcal{J})$

$$
\begin{array}{ccccccc}
\mathtt{cov}^*(\mathcal{I}) & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{I},\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{I},\mathcal{J}) \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\mathfrak{p} & \longrightarrow & \mathfrak{p}_{\text{1-1}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) & \longrightarrow & \mathfrak{p}_{\mathrm{K}}(\mathcal{J})
\end{array}
$$

Theorem (P. Borodulin-Nadzieja and B. Farkas 2012)

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH** *the following hold.*

(1) *There is a filter $\mathcal{F}$ with $\mathfrak{p}_{\text{1-1}}(\mathcal{F}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{F}) = \mathfrak{p}_{\mathrm{K}}(\mathcal{F}) = \omega_2$.*

(2) *There is a meager filter $\mathcal{G}$ with $\mathfrak{p}_{\text{1-1}}(\mathcal{G}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{G}) = \omega_1$ and $\mathfrak{p}_{\mathrm{K}}(\mathcal{G}) = \omega_2$.*

(3) $\mathfrak{p}_{\text{1-1}}(\mathcal{J}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) = \omega_1$ *for every F_σ ideal $\mathcal{J}$ and every analytic P-ideal $\mathcal{J}$.*

Problem 2: covering counterpart of $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

X is an $\mathrm{S}_1(\Omega^{\mathtt{ct}}, \Gamma)$-space if and only if X has $\binom{\Omega^{\mathtt{ct}}}{\Gamma}$.

Question (B. Tsaban ESTC 2019, Vienna)

Is it true that X is an $\mathrm{S}_1(\Omega^{\mathtt{ct}}, \mathcal{I}\text{-}\Gamma)$-space if and only if X has $\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{I}\text{-}\Gamma\end{smallmatrix}\right]$?

P. Borodulin-Nadzieja and B. Farkas 2012

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH**:

- there is a meager ideal $\mathcal{I}$,
- there is a set of reals A of size ω_1,
- A has $\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{I}\text{-}\Gamma\end{smallmatrix}\right]$,
- A is not an $\mathrm{S}_1(\Omega^{\mathtt{ct}}, \mathcal{I}\text{-}\Gamma)$-space.

Problem 2: covering counterpart of $\mathcal{I}$-Fréchet-Urysohn property

$$\boxed{\mathtt{non}(\mathrm{S}_1(\Omega^{\mathtt{ct}}, \mathcal{J}\text{-}\Gamma)) = \lambda(*, \mathcal{J})}$$

$$\boxed{\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}} \\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J})}$$

Problem 2: covering counterpart of $\mathcal{I}$-Fréchet-Urysohn property

$$\boxed{\mathtt{non}(\mathrm{S}_1(\Omega^{\mathtt{ct}}, \mathcal{J}\text{-}\Gamma)) = \lambda(*, \mathcal{J})}$$

$$\boxed{\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}} \\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J})}$$

A sequence $\langle V_n : n \in \omega \rangle$ of open subsets of X such that $V_n \neq X$ is an ω-cover if for every $a \in [X]^{<\omega}$ there is n such that $a \subseteq V_n$. $\Omega^{\mathtt{ct}}$

Problem 2: covering counterpart of $\mathcal{I}$-Fréchet-Urysohn property

$$\boxed{\mathtt{non}(\mathrm{S}_1(\Omega^{\mathtt{ct}}, \mathcal{J}\text{-}\Gamma)) = \lambda(*, \mathcal{J})}$$

$$\boxed{\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}} \\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J})}$$

A sequence $\langle V_n : n \in \omega \rangle$ of open subsets of X such that $V_n \neq X$ is an ω-cover if for every $a \in [X]^{<\omega}$ there is n such that $a \subseteq V_n$. $\Omega^{\mathtt{ct}}$

Proposition

Let X be a topological space. If $\mathcal{J}$ has Baire property then

$$X \textit{ is an } \mathrm{S}_1(\Omega^{\mathtt{ct}}, \mathcal{J}\text{-}\Gamma)\textit{-space if and only if } X \textit{ is an } \mathrm{S}_1(\Omega^{\mathtt{ct}}, \Gamma)\textit{-space.}$$

Problem 2: covering counterpart of $\mathcal{I}$-Fréchet-Urysohn property

$$\boxed{\mathtt{non}(\mathrm{S}_1(\Omega^{\mathtt{ct}}, \mathcal{J}\text{-}\Gamma)) = \lambda(*, \mathcal{J})}$$

$$\boxed{\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}} \\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J})}$$

A sequence $\langle V_n : n \in \omega \rangle$ of open subsets of X such that $V_n \neq X$ is an ω-cover if for every $a \in [X]^{<\omega}$ there is n such that $a \subseteq V_n$. $\Omega^{\mathtt{ct}}$

Proposition

Let X be a topological space. If $\mathcal{J}$ has Baire property then

X is an $\mathrm{S}_1(\Omega^{\mathtt{ct}}, \mathcal{J}\text{-}\Gamma)$-space if and only if X is an $\mathrm{S}_1(\Omega^{\mathtt{ct}}, \Gamma)$-space.

Theorem (P. Borodulin-Nadzieja and B. Farkas 2012)

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH** *there is a meager ideal $\mathcal{J}$ such that $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) = \omega_2$.*

Problem 1: countable $\mathcal{I}$-Fréchet-Urysohn property

Theorem (J. Gerlits and Zs. Nagy 1982)

$C_p(X)$ has countable Fréchet-Urysohn property if and only if $C_p(X)$ is an $\mathrm{S}_1(\Omega_{\mathbf{0}}^{\mathtt{ct}}, \Gamma_{\mathbf{0}})$-space countable covers.

P. Borodulin-Nadzieja and B. Farkas 2012

$\mathcal{I}$-Fréchet-Urysohn property

In a Cohen forcing model adding ω_2 many Cohen reals to a model of **ZFC**$+$**GCH**:

- there is a meager ideal $\mathcal{I}$,
- there is a set of reals A of size ω_1,
- $C_p(A)$ has countable $\mathcal{I}$-Fréchet-Urysohn property,
- $C_p(A)$ is not an $\mathrm{S}_1(\Omega_{\mathbf{0}}^{\mathtt{ct}}, \mathcal{I}\text{-}\Gamma_{\mathbf{0}})$-space.

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ *and* $\mathfrak{p}_{1\text{-}1}(\mathcal{J})$*?*

$$\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\mathrm{KB}}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{1\text{-}1}) = \mathfrak{p}_{1\text{-}1}(\mathcal{J})$$

Different repetitions of elements (infinitely many, finitely many, none) in the enumeration of sequence.

Similarly for functional versions.

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Is $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$ *for each analytic (*P*-)ideal* $\mathcal{J}$*?*

Proposition

If $\mathcal{J}$ *is a meager* P*-ideal then* $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$*.*

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ *and* $\mathfrak{p}_{\text{1-1}}(\mathcal{J})$*?*

$$\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\mathrm{KB}}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathtt{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\text{1-1}}) = \mathfrak{p}_{\text{1-1}}(\mathcal{J})$$

Different repetitions of elements (infinitely many, finitely many, none) in the enumeration of sequence.

Similarly for functional versions.

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ *and* $\mathfrak{p}_{1\text{-}1}(\mathcal{J})$*?*

$$\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\mathrm{KB}}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{1\text{-}1}) = \mathfrak{p}_{1\text{-}1}(\mathcal{J})$$

Different repetitions of elements (infinitely many, finitely many, none) in the enumeration of sequence.

Similarly for functional versions.

- X is an $[\Omega^{\mathrm{ct}}, \mathcal{J}\text{-}\Gamma]_{\square}$-space if for every ω-cover $\langle V_n : n \in \omega \rangle$ there is $\square$-function $\varphi \in {}^{\omega}\omega$ such that $\langle V_{\varphi(m)} : m \in \omega \rangle$ is a $\mathcal{J}$-γ-cover.

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Do there exist ~~reasonable~~ topological characterizations of $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J})$ and $\mathfrak{p}_{1\text{-}1}(\mathcal{J})$?

$$\mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]) = \mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{\mathrm{KB}}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \qquad \mathtt{non}(\left[\begin{smallmatrix}\Omega^{\mathrm{ct}}\\ \mathcal{J}\text{-}\Gamma\end{smallmatrix}\right]_{1\text{-}1}) = \mathfrak{p}_{1\text{-}1}(\mathcal{J})$$

Different repetitions of elements (infinitely many, finitely many, none) in the enumeration of sequence.

Similarly for functional versions.

- X is an $[\Omega^{\mathrm{ct}}, \mathcal{J}\text{-}\Gamma]_{\square}$-space if for every ω-cover $\langle V_n : n \in \omega \rangle$ there is $\square$-function $\varphi \in {}^{\omega}\omega$ such that $\langle V_{\varphi(m)} : m \in \omega \rangle$ is a $\mathcal{J}$-γ-cover.

Observation

If X is a topological space then the following are equivalent.

(a) *X is an $[\Omega^{\mathrm{ct}}, \mathcal{J}\text{-}\Gamma]_{\square}$-space.*

(b) *For every family $\mathcal{V}$ which forms a countable open ω-cover there is a $\mathcal{J}$-γ-cover $\langle V_m : m \in \omega \rangle$ such that $V_m \in \mathcal{V}$ and a set V_m may be repeated $\square$-many times in the enumeration.*

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Is $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$ *for each analytic (*P*-)ideal* $\mathcal{J}$*?*

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Is $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$ *for each analytic (*P*-)ideal* $\mathcal{J}$*?*

Proposition (P. Borodulin-Nadzieja and B. Farkas 2012)

If $\mathcal{J}$ *is meager then* $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \leq \mathfrak{b}$*.*

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Is $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$ *for each analytic (*P*-)ideal* $\mathcal{J}$*?*

Proposition (P. Borodulin-Nadzieja and B. Farkas 2012)

If $\mathcal{J}$ *is meager then* $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \leq \mathfrak{b}$*.*

Proposition (M. Repický 2018)

If $\mathcal{J}$ *is a* P*-ideal then* $\mathfrak{p}_{\mathrm{K}}(\mathcal{I}, \mathcal{J}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{I}, \mathcal{J})$*.*

Problem 3: pseudointersection numbers

Problem (P. Borodulin-Nadzieja and B. Farkas 2012)

Is $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$ *for each analytic (*P*-)ideal* $\mathcal{J}$*?*

Proposition (P. Borodulin-Nadzieja and B. Farkas 2012)

If $\mathcal{J}$ *is meager then* $\mathfrak{p}_{\mathrm{KB}}(\mathcal{J}) \leq \mathfrak{b}$*.*

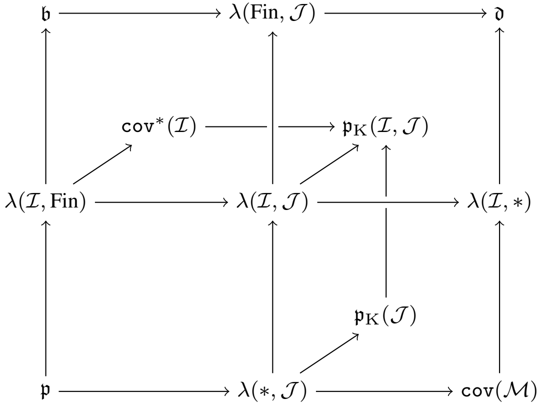
Proposition (M. Repický 2018)

If $\mathcal{J}$ *is a* P*-ideal then* $\mathfrak{p}_{\mathrm{K}}(\mathcal{I}, \mathcal{J}) = \mathfrak{p}_{\mathrm{KB}}(\mathcal{I}, \mathcal{J})$*.*

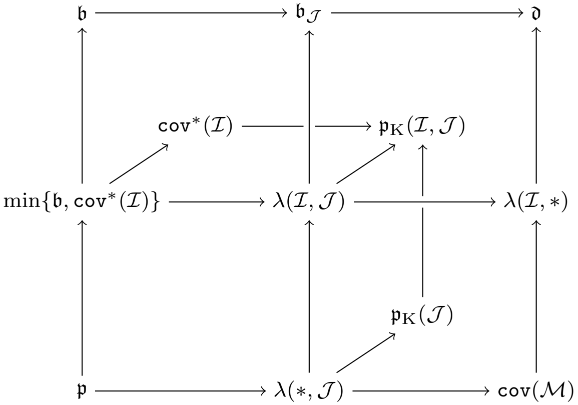
Corollary

If $\mathcal{J}$ *is a meager* P*-ideal then* $\mathfrak{p}_{\mathrm{K}}(\mathcal{J}) \leq \mathfrak{b}$*.*

Critical cardinalities

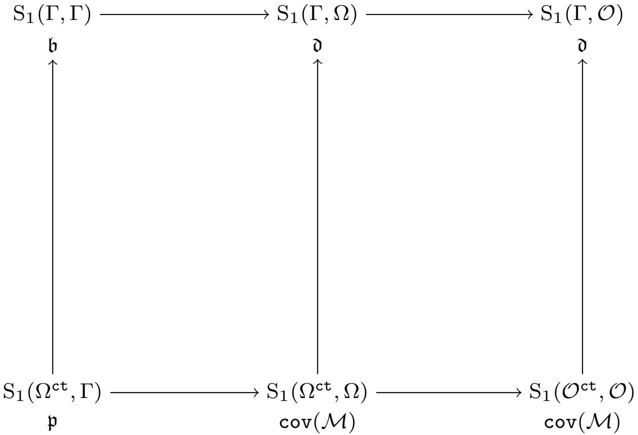


Critical cardinalities

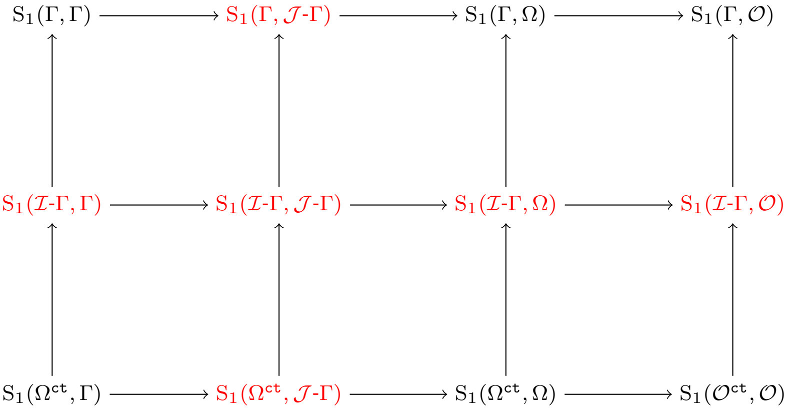


Principle $\mathrm{S}_1(\mathcal{P},\mathcal{R})$ and corresponding critical cardinality

Just W., Miller A.W., Scheepers M. and Szeptycki P.J., *Combinatorics of open covers* II, Topology Appl. **73** (1996), 241–266.



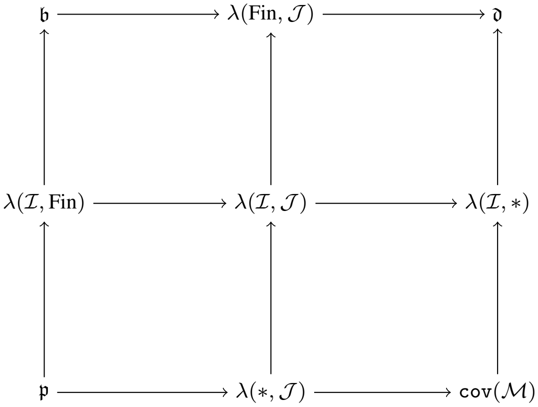
Principle $\mathrm{S}_1(\mathcal{P},\mathcal{R})$ and ideal covers



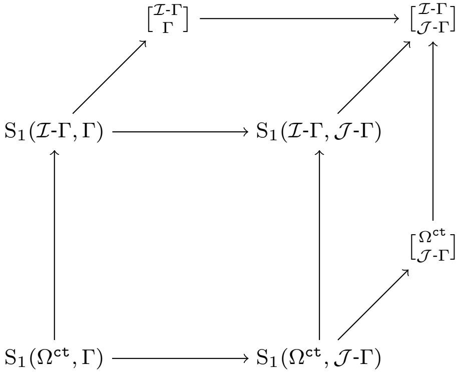
Principle $\mathrm{S}_1(\mathcal{P},\mathcal{R})$ and corresponding critical cardinality

$$
\begin{array}{ccccccc}
\mathrm{S}_1(\Gamma,\Gamma) & \longrightarrow & \mathrm{S}_1(\Gamma,\mathcal{J}\text{-}\Gamma) & \longrightarrow & \mathrm{S}_1(\Gamma,\Omega) & \longrightarrow & \mathrm{S}_1(\Gamma,\mathcal{O}) \\
\mathfrak{b} & & \lambda(\mathrm{Fin},\mathcal{J}) & & \mathfrak{d} & & \mathfrak{d} \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\mathrm{S}_1(\mathcal{I}\text{-}\Gamma,\Gamma) & \longrightarrow & \mathrm{S}_1(\mathcal{I}\text{-}\Gamma,\mathcal{J}\text{-}\Gamma) & \longrightarrow & \mathrm{S}_1(\mathcal{I}\text{-}\Gamma,\Omega) & \longrightarrow & \mathrm{S}_1(\mathcal{I}\text{-}\Gamma,\mathcal{O}) \\
\lambda(\mathcal{I},\mathrm{Fin}) & & \lambda(\mathcal{I},\mathcal{J}) & & \lambda(\mathcal{I},*) & & \lambda(\mathcal{I},*) \\
\uparrow & & \uparrow & & \uparrow & & \uparrow \\
\mathrm{S}_1(\Omega^{\mathtt{ct}},\Gamma) & \longrightarrow & \mathrm{S}_1(\Omega^{\mathtt{ct}},\mathcal{J}\text{-}\Gamma) & \longrightarrow & \mathrm{S}_1(\Omega^{\mathtt{ct}},\Omega) & \longrightarrow & \mathrm{S}_1(\mathcal{O}^{\mathtt{ct}},\mathcal{O}) \\
\mathfrak{p} & & \lambda(*,\mathcal{J}) & & \mathtt{cov}(\mathcal{M}) & & \mathtt{cov}(\mathcal{M})
\end{array}
$$

Critical cardinalities



Subsequence schema



Sample values

V. Šottová and J.Š. 2019, V. Šottová 2019

$$\boxed{\lambda(\mathrm{Fin}, \mathrm{Fin}) = \mathfrak{b}}$$

- $\lambda(\mathtt{S}, \mathrm{Fin}) = \lambda(\mathtt{S}, \mathtt{S}) = \min\{\mathfrak{b}, \mathtt{non}(\mathcal{N})\}$
- $\lambda(\mathtt{nwd}, \mathrm{Fin}) = \lambda(\mathtt{nwd}, \mathtt{nwd}) = \mathtt{add}(\mathcal{M})$
- $\lambda(\mathcal{R}, \mathcal{J}) = \lambda(\mathrm{Fin}, \mathcal{J}) = \mathfrak{b}_{\mathcal{J}}$
- $\lambda(\mathtt{conv}, \mathcal{J}) = \lambda(\mathrm{Fin}, \mathcal{J}) = \mathfrak{b}_{\mathcal{J}}$
- there is $\mathcal{U}$ such that $\lambda(\mathcal{U}, \mathrm{Fin}) = \mathfrak{p}$

Borodulin–Nadzieja P. and Farkas B., *Cardinal coefficients associated to certain orders on ideals*, Arch. Math. Logic **51** (2012), 187–202.

Brendle J. and Shelah S., *Ultrafilters on ω–their ideals and their cardinal characteristics,* Trans. Amer. Math. Soc. **351** (1999), 2643-–2674.

Gerlits J. and Nagy Zs., *Some properties of $C_p(X)$, I,* Topology Appl. **14** (1982), 151–161.

Hernández F. and Hrušák M., *Cardinal invariants of analytic P-ideals,* Canad.J.Math. **59** (2007), 575–595.

Hrušák M., *Combinatorics of filters and ideals,* Contemp. Math. **533** (2011), 29–69.

Das P., *Certain types of open covers and selection principles using ideals,* Houston J. Math. **39** (2013), 637–650.

Nyikos P., *Special ultrafilters and cofinal subsets of $({}^{\omega}\omega, <^*)$,* preprint.

Repický M., *Spaces not distinguishing ideal convergences of real-valued functions*, preprint.

Rothberger F., *On some problems of Hausdorff and of Sierpiński*, Fund. Math. **35** (1948), 29–46.

Šottová V., *Cardinal invariant $\lambda(\mathcal{S}, \mathcal{J})$,* GEYSER MATH. CASS. **1** (2019), 64–72.

Šottová V. and Šupina J., *Principle $\mathrm{S}_1(\mathcal{P}, \mathcal{R})$: ideals and functions,* Topology Appl. **258** (2019), 282–304.

Thanks for Your attention!

A family $\mathcal{K} \subseteq \mathcal{P}(\omega)$ is called an ideal if

a) $B \in \mathcal{K}$ for any $B \subseteq A \in \mathcal{K}$,
b) $A \cup B \in \mathcal{K}$ for any $A, B \in \mathcal{K}$,
c) $\mathrm{Fin} = [\omega]^{<\omega} \subseteq \mathcal{K}$,
d) $\omega \notin \mathcal{K}$.

$\mathcal{I}$, $\mathcal{J}$, $\mathcal{K}$ are ideals in the following.

$\mathcal{K} \subseteq \mathcal{P}(\omega)$ $\qquad$ $\mathcal{K}^+ = \mathcal{P}(\omega) \setminus \mathcal{K}$

$\mathcal{A} \subseteq \mathcal{P}(\omega)$ $\qquad$ $\mathcal{A}^d = \{A \subseteq \omega : \ \omega \setminus A \in \mathcal{A}\}$

$\mathcal{F} \subseteq \mathcal{P}(\omega)$ is a filter if $\mathcal{F}^d$ is an ideal.

A maximal filter $\mathcal{U} \subseteq \mathcal{P}(\omega)$ is called an ultrafilter.

Ideal covers

A sequence $\langle V_n : n \in \omega \rangle$ of open subsets of X such that $V_n \neq X$ is

- `cover` if for every $x \in X$ there is n such that $x \in V_n$.
- ω-`cover` if for every $a \in [X]^{<\omega}$ there is n such that $a \subseteq X$.
- $\mathcal{I}$-γ-`cover` if $\{n : x \notin V_n\} \in \mathcal{I}$ for every $x \in V_n$.
- γ-`cover` if $\{n : x \notin V_n\}$ is finite for every $x \in X$.

$$\boxed{\Gamma \subseteq \mathcal{I}\text{-}\Gamma \subseteq \Omega \subseteq \mathcal{O}}$$